MODEL ELICITING ACTIVITIES AS A POWERFUL TOOL TO EXPAND COLLABORATIVE WORK OPPORTUNITIES FOR UNDERGRADUATE STUDENTS

Luis E. Montero-Moguel
The University of Texas at San Antonio
luis.monteromoguel@utsa.edu

Verónica Vargas-Alejo
University of Guadalajara
veronica.vargas@academicos.udg.mx

Guadalupe Carmona
The University of Texas at San Antonio
guadalupe.carmona@utsa.edu

Dinorah Méndez Huerta
University of Guadalajara
dinorah.mendez7682@alumnos.udg.mx

This research contributes to the need to identify and expand learning environments that encourage undergraduate students to develop collaborative work skills and apply their classroom knowledge to solve real-world problems. Using qualitative methods, we examine the effects of the interaction between two teams of students when solving a Model-eliciting activity, based on the theoretical framework of the Models and Modeling Perspective. Our analysis shows that the students had two opportunities for interaction, within the team and between the modeling teams. Through these interactions, the students refined their models in three directions: mathematical knowledge, interpretation and modeling of the phenomenon, and mathematical representations. The findings of this study emphasize the significant value of Model-eliciting activities in enhancing students' collaboration and modeling abilities.

Keywords: Modeling, Undergraduate education, Precalculus, Model eliciting activities.

Introduction

Learning opportunities for higher education should help students develop, in-depth knowledge of the disciplines, as well as skills that allow them to work collaboratively to solve the problems of 21st century (National Academies of Sciences, Engineering, and Medicine, NASEM, 2018a; National Science Foundation, 2016). In addition, collaborative work favors the cognitive growth of students (NASEM, 2018b). However, employers recognize that students graduating from higher education lack communication and teamwork skills (Felder, 2021). Thus, it is necessary to broaden research around the search for learning environments that encourage the collaborative work of undergraduate students to allow them to deepen their knowledge of the disciplines in contexts close to those they will face in their professional lives.

One of the ways to addresses the preparation of undergraduate students in real-life contexts is through mathematical modeling because "it connects mathematics to real-world questions through the cyclic process of receiving a non-mathematized task, applying mathematics to better understand the phenomenon and then circling back to the real-world questions to arrive at solutions" (Hjalmarson et al., 2020, p. 225). This connection between mathematics and real-life situations is central, the mathematical modeling activities should propose that students do not focus on modeling the data, but that the modeling activities encourage students to delve into the interpretation and modeling of the phenomenon (Stillman & Brown, 2021). Zawojewski et al. (2003) point out that groups of elementary students who participate in model-eliciting activities (MEA) "learn that together they are quite powerful, that they need to listen to each other, and that they can indeed solve these complex problems, they develop a clearer, more stable idea of how modeling activities can be accomplished" (p. 357). However, it remains uncertain whether these

types of interactions are similarly generated within undergraduate student modeling teams. We believe that the research by Zawojewski et al. (2003) should be extended to understand if the MEAs contribute in the same sense in undergraduate students.

Given the relevance of providing undergraduate students with skills that allow them to collaboratively solve real-life problems, this qualitative study aims to describe the effect generated in the modeling process by the interaction of two teams of undergraduate business students while participating in a modeling activity. The research questions that guided this study were, (1) to what extent does the interaction between the student modeling teams favor the modification of their initial models when solving a MEA? and (2) what opportunities for interaction does the MEA generate?

Theoretical Framework

This study was based on the Models and Modeling perspective (MMP). According to the MMP, students work individually or in teams to build models that allow them to describe, explain, and predict intentionally designed situations in real contexts (Ärlebäck & Doerr, 2018). According to the MMP, the models are conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently.

A mathematical model focuses on structural characteristics (rather than, for example,physical or musical characteristics) of the relevant systems (Lesh & Doerr, 2003, p. 10). These intentionally constructed situations are called MEAs, they are built under six principles: personal meaningfulness or reality principle, model construction, self-evaluation, model externalization or model-documentation, simple prototype, and model generalization (Lesh et al., 2000). MEAs encourage students to "develop modeling cycles, also called iterative modeling cycles, which are interpretations (or conceptual systems) of problem situations" (Montero-Moguel & Vargas-Alejo, 2022, p. 218). When students solve the MEAs in small groups (between two and three students), they face complex problems that require multiple cycles of interpretation and refinement, collaborative work and communication are a fundamental part for students to expand their conceptual system (Zawojewski & Carmona, 2001). The conceptual system tends to be expressed externally through a variety of representations, for example, written symbols, graphs, spoken language, diagrams, tables, and equations (Lesh & Doerr, 2003). The collaborative work of elementary students when solving MEA encourages them to "learn the power of representing their ideas externally (e.g., drawing a picture) to communicate with their peers, and the power of taking different points of view as they experience the process of comparing, contrasting, and reconciling their initial interpretations" (Zawojewski et al., 2003, p. 341).

Methods

This qualitative study involved two teams of three freshman undergraduate business students at a university in Mexico. Both teams worked in the same classroom and had access to computers. The students were studying the subject of mathematics applied to business. Prior to the study, the students had not addressed the issue of functions during their professional career. For this study, we constructed the MEA called "Yogurt is breaking investment barriers" (Yogurt MEA), based on the six principles of MEA construction (Lesh et al., 2000). Some big math ideas built into the design of Yogurt MEA are: exponential function, logistic function, variation, growth, and rate of change. The MEA was divided into three parts, the first a journalistic note in the context of health care, the consumption of healthy yogurt and the high profits of yogurt companies. The second section contained questions about the context of the journalistic note. The third part, the problem

situation, included a video (https://youtu.be/4y6V_6HFf_E) containing images of seven days of culturing yogurt kefir grains, the images showing the number of yogurt kefir grains growth and the temperature on each day. Under the context of the creation of a yogurt company, the MEA encourages students to build models to describe the growth of yogurt kefir grains that allow them to start the production of a Yogurt company. The implementation was conducted in four phases.

1. Warm-up activity. Reading of the journalistic note, warm-up questions and discussion of the context. The participation in this phase was group.
2. Resolution of the problem situation. Modeling of the Yogurt MEA problem situation. The participation in this phase was in teams.
3. Plenary. Presentation of models of both teams and discussion. The participation in this phase was group.
4. Resolution of the problem situation. Second modeling of the Yogurt MEA problem situation. The participation in this phase was in teams.

Data collected for analysis came from researchers' detailed review of implementation video recordings and transcripts, group discussion, team model presentation, the models built by the students (using Word, Excel and PDF documents), and the researcher's reports. Data was collected during the implementation of the MEA. All students' responses were in Spanish. Thus, translation of select excerpts of the transcripts are made available in English for this manuscript.

In order to identify and compare the models built by the teams, this qualitative study developed a first cycle of *a priori* coding for each team separately from the modeling cycles of the teams. We chose *a priori* coding because it "is appropriate for qualitative studies in disciplines with pre-established and field-tested coding systems if the researcher's goals harmonize with the protocol's outcomes" (Saldaña, 2016, p. 175). In this study, our coding and analysis system is based on the MMP. Since to the relevance of analyzing the students' interpretations, we code the data in three directions: a) *mathematical knowledge* (mathematical concepts used to describe and predict the phenomenon), b) *modeling of the phenomenon* (students focus on modeling the characteristics of the phenomenon not on modeling the data), and c) *representations* (the mathematical representations used to describe the model, e.g., graphs, tables). The results were discussed among the researchers until consensus was reached based on the theoretical framework. For the second cycle of analysis, we analyzed the differences and coincidences between the coded results of both teams. The data was analyzed using the NVivo 12 software, which allowed the association and identification of patterns in the models of each team and between the models of both teams. The results were organized into two modeling cycles, which are presented in the following section.

Analysis and discussion

The analysis focused on two team modeling cycles, one prior to the group discussion and the second after the group discussion. In the first cycle both teams worked independently to build their initial models; subsequently they presented and discussed their models as a group. The analysis allowed us to identify that, in the first cycle, the models of both teams were based on different mathematical concepts, they used different mathematical representations to describe their model, and even the interpretation and control of the phenomenon was different in both models. The analysis of the second cycle allowed us to identify that both teams expanded and refined their models. We identified that the students' models shared coincidences in mathematical concepts, understanding and control of phenomena, and mathematical representations. Both cycles built by the teams are described below.

First Modeling Cycle

For both teams, the work during the construction process of the initial models was a complex task that required multiple cycles of interpretation; collaborative work within the team was essential to solve the MEA.

Team 1 Model. Team 1's interpretation of the phenomenon led the students to identify that it is relevant that yogurt kefir grains are grown in jars. Based on this interpretation, the students based their model on the growth of yogurt kefir grains in different jars. The students represented in a table the data extracted from the video on the growth of yogurt kefir grains in milliliters and based on this table they built three more. They used the mathematical concept of proportionality to identify the initial and final percentage growth of yogurt kefir grains after one month of production. The students' model included tabular and verbal representations (Figure 1). During the presentation of their model, Team 1 mentioned the following.

Team 1: Aquí pusimos la primera semana cómo fue creciendo de 40 a 200 y a la segunda semana, para que vean el crecimiento, ya lo separamos el de 200 en cinco frascos y en lugar de hacer 200 al final llega mil. Después continuamos, dividimos los mil entre 4 frascos y salieron 25. Pusimos en cada frasco 40 mililitros y tuvimos un crecimiento de 12 en total. En un mes tuvimos un crecimiento de 625%. [Here we describe in the first week how it grew from 40 to 200. The second week, so that you can see the growth, we separated the 200 into five jars and instead of making 200 at the end the total is one thousand. Then we continued, we divided the thousand between 4 bottles and 25 came out. We put 40 milliliters in each bottle, and we had a growth of 12 in total. In one month, we had a growth of 625%].

Kéfir Healthy
Presente

Por medio de la presente, le comunico los resultados obtenidos de la investigación realizada al comportamiento de crecimiento de los búlgaros. Como mostramos a continuación tuvimos un crecimiento del 625% en 4 semanas. También es importante mencionar que la temperatura se mantuvo en un promedio de 20ºC.

Semana 1			
dia	frascos	mililitros	Total mL
1	1	40	40
2	1	50	50
3	1	70	70
4	1	90	90
5	1	120	120
6	1	150	150
7	1	200	200

Semana 2			
dia	frascos	mililitros	Total mL
1	5	40	200
2	5	50	250
3	5	70	350
4	5	90	450
5	5	120	600
6	5	150	750
7	5	200	1000

Semana 3			
dia	frascos	mililitros	Total mL
1	25	40	1000
2	25	50	1250
3	25	70	1750
4	25	90	2250
5	25	120	3000
6	25	150	3750
7	25	200	5000

Semana 4			
dia	frascos	mililitros	Total mL
1	125	40	5000
2	125	50	6250
3	125	70	8750
4	125	90	11250
5	125	120	15000
6	125	150	18750
7	125	200	25000

Dependiendo de la cantidad de mililitros de búlgaros necesarios para el proceso del yogurt casero y la cantidad que ustedes necesitan para ofertar, se podría determinar cuando es el mejor momento para empezar con la venta.

Figure 1: Letter from Team 1 in the first modeling cycle

Team 2 Model. Based on the context, Team 2 identified that setting the temperature value was a factor that improved the growth of dairy yogurt kefir grains. The students' model was based on

the mathematical concept of the average growth rate. The students built a table containing the days, the amount of yogurt kefir grains and the temperature. With these data they calculated the growth rate that served them to describe and predict the growth of dairy yogurt kefir grains. The students included tabular, graphic, and verbal representations in their model (Figure 2). During the presentation of their model, Team 2 mentioned the following.

> Team 2: Sacamos los porcentajes. La variación de porcentaje del día uno al día dos. Entonces me daba un resultado de 20%. Y así sucesivamente. Calculando el porcentaje del crecimiento. […]. Yo dije mantenemos siempre la temperatura, sacamos un promedio en el porcentaje del crecimiento que me daban a 21 grados y encontramos un promedio de 33.33%. Entonces, si yo mantengo siempre mi temperatura 21 grados porcentualmente más o menos voy a tener un crecimiento del 33%, a lo que del día uno al día cuatro yo iba a tener un crecimiento de 4570 más de mi colonia de bacterias. [We got the percentages. The percentage change from day one to day two. Then it gave me a result of 20%, and so on. Calculating the percentage of growth. […]. I said we always maintain the temperature; we took the average of the percentage of growth at 21 degrees and we found an average of 33.33%. So, if I always maintain my temperature 21 degrees, I will have a growth of approximately 33%, so from day one to day four I would have a growth of 4570 more for my colony of bacteria].

Para: Corporativo Kéfir Healthy

Por medio de la presente carta, queremos informarle sobre el crecimiento de la cantidad de búlgaros.
Los resultados arrojan que
El Día 1 al Día 2 tenemos un crecimiento del 20% manteniendo una temperatura de 20 grados centígrados.
Del Día 2 al Día 3 se obtiene un crecimiento del 40 % manteniéndolo a temperatura a 21 grados centígrados
Del Día 3 al Día 4 se obtiene un crecimiento del 40 % manteniéndolo a 21 grados centígrados.
DEL Día 4 al Día 5 tenemos de porcentaje del 40% a 30 % con la disminución de temperatura de 20 grados centígrados.
Del Día 5 al Día 6 sigue disminuyendo a 25% manteniendo a 20 grados centígrados.
Del día 6 al Día 7 aumenta a 33% aumentando la temperatura a 21 grados centígrados.

En conclusión, manteniendo la temperatura a 21 grados centígrados tenemos un mejor crecimiento en promedio de 33%.
Si mantenemos la temperatura en 21 grados centígrados del día 1 con 40 ml de búlgaros al dia 31 del mes tendremos un crecimiento de 4576 veces más que el día 1

dia	ml	temperatura	%	promedio de crecimiento		
1	40.00	20				4,576.09
2	50.00	21	20			
3	70.00	21	40	33		
4	90.00	21	40	122.5		
5	120.00	20	30			
6	150.00	20	25			
7	200.00	21	33			
8	260.00	21	30			
	345.00	21	33			
	458.85	21	33			
	610.27	21	33			
	811.65	21	33			
	1,079.50	21	33			
	1,435.74	21	33			
	1,909.54	21	33			
	2,539.68	21	33			
	3,377.78	21	33			
	4,492.45	21	33			
	5,974.96	21	33			
	7,946.70	21	33			
	10,569.11	21	33			
	14,056.92	21	33			

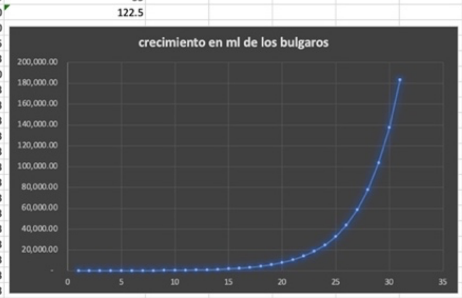

Figure 2: Letter from Team 2 in the first modeling cycle

Observations. In the first cycle, both teams' models described, predicted, and even controlled the growth of dairy yogurt kefir grains. Both teams focused on modeling the phenomenon, however each emphasized different controls to enhance the growth of the yogurt kefir grains. Team 1 focused on controlling the culture in jars and Team 2 on controlling the temperature. The mathematical concepts underlying the models of each team were different. Team 1 used proportionality and Team 2 the average growth rate associated with exponential behavior. Both teams constructed tables and verbally described their models; in addition, Team 2 included a graphic representation.

Second Modeling Cycle

After the first cycle, the teams presented their models, interacted with questions, and reflected on their models and the other team's models. Subsequently, the teams worked on a second modeling cycle to modify their models from the first cycle.

Team 1 Model. Team 1 reflected on the phenomenon, they continued to consider important the cultivation of yogurt kefir grains in jars and added in their interpretation the importance of controlling the temperature. To describe and predict the growth of yogurt kefir grains, Team 1 calculated the average growth rate for the first seven days (33%) and used this rate to exponentially describe the growth of yogurt kefir grains. Team 1, which had initially included verbal tabular representations in its initial model, added graphical and algebraic representations to its second model (Figure 3). This modification coincides with the findings of Lesh and Doerr (2003), regarding the fact that students generate modifications in their representations during the modeling process.

Figure 3: Graphic included in the second model of Team 1

Team 2 Model. Team 2 maintained the idea of controlling the phenomenon of the growth of yogurt kefir grains by setting the controlled temperature, also included descriptions to denote that the culture would be carried out in jars to favor the growth of the bacterium. They argued that since the culture would be in jars, then they could remove half of the yogurt kefir grains on day five to improve growth. This consideration impacted their mathematical analysis, although the model continued to be based on the average growth rate analysis, the team described growth before five days and after those five days considering an average growth rate of 33% for both. Students

included tabular, verbal, and graphic representations. Team 2 described the following in their second model:

> Team 2: Pero si el día 5 tomamos la mitad de los búlgaros y los colocamos en otro frasco y empezamos a crear un ambiente favorable para los mismos, al final del mes tendremos una mayor cantidad de la bacteria para iniciar el proceso de producción y tienen un mejor rendimiento en su multiplicación. Que también se pone en etapa de prueba el estar dividiendo en frascos. Y seguir aumentando la cantidad en producción. [But if on the 5th we take half of the yogurt kefir grains and place them in another jar and begin to create a favorable environment for them, at the end of the month we will have a greater number of the bacteria to start the production process and have a better yield in their multiplication. That dividing more jars is also put in the test stage. And keep increasing the quantity in production].

Observations. In the second cycle both teams refined their initial models. The two teams deepened their knowledge of the phenomenon and control of variables. Both teams proposed two ways to control variables, through temperature and culture in jars. Based on the characteristics of the phenomenon, both teams based their models on the calculation of the average daily growth rate associated with the mathematical concept of exponential function. To build their models, the teams used verbal, tabular, and graphical representations.

Conclusions

In response to the research question: *to what extent does the interaction between the student modeling teams favor the modification of their initial models when solving a MEA?* Our analysis identified the following results. Through the interaction of the teams after their first MEA modeling cycle, both teams refined their models in different directions: mathematical knowledge, interpretation and modeling of the phenomenon, and representations included in the models. *Mathematical knowledge*, Team 1, which initially built a model based on proportionality analysis, refined its model to incorporate similar characteristics to Team 2's model, which used an analysis supported by the average growth rate. *Interpretation and modeling of the phenomenon*, Team 2 initially focused on the growth of yogurt kefir grains considering temperature control but without considering the container, they included in their second model descriptions similar to those of Team 1's model, based on the importance of having bacteria in jars to improve reproduction. *Representations included in the models*, Team 1, which had initially included tabular and verbal representations, included in its second model a graph similar to that of the initial model of Team 1 and additionally included an algebraic representation.

Regarding the second research question: *what opportunities for interaction does the MEA generate?* This study found that the implementation of this MEA gave the students two different levels of interaction, the first is the work *within* the modeling team and the second, the interaction *between* modeling teams. At the first level of interaction, it is noteworthy how in the process of solving the MEA, the interaction between the members of each team, their mathematical knowledge and their interpretation of the phenomenon allowed the development of an iterative process among the team members to face a complex problem. The students' model allowed them to describe, predict, and interpret the MEA phenomenon with the customer in mind. Our findings align with the research conducted by Zawojewski et al. (2003) at the elementary level, because, undergraduate students can learn that teamwork allows them to compare, contrast, and extend their initial interpretations. Additionally, we concur with Zawojewski and Carmona (2001) and Zawojewski et al. (2003) regarding that when students work in small teams, they recognize that

working together allows them to face complex problems. Moreover, our findings support the perspective put forth by NASEN (2018b) that students' cognitive growth is fostered through collaborative work. At the second level of interaction, our findings reveal that the students did not discard their initial models, but rather, the interaction with the other team allowed them to expand and refine their models. The MEA, therefore, provides students with the opportunity to share, evaluate, reflect, and modify their own proposals based on interaction with a team with different ideas.

These findings contribute to the needs identified by NASEM (2018a) regarding generating learning environments where undergraduate students improve their collaborative work skills as they deepen the concepts of the disciplines, with the aim of facing the real problems of our century. Future research could explore how to integrate these findings into teaching practices to improve undergraduate students' collaborative work skills and problem-solving abilities.

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